**Lecture 4 (Options Markets) Assignment, MTH 9865**

Due start of class, October 6, 2015

**Question 1 (4 marks)**

The current time is Wednesday at 1pm and you see the overnight implied volatility (for 10am expiration on Thursday) trading at 9%. The FX markets are open for trading every hour between now and tomorrow at 10am.

The Federal Reserve Chairwoman is speaking about the economy from 2-3pm, and that event adds an extra 0.5 trading days worth of variance on top of the usual variance for that time period.

What should the overnight implied volatility be at 3pm, all else being equal?

The strategy here: calculate the trading time vol from the implied vol mark, and assume that trading time vol is unchanged through the event.

At 1pm, you see the implied volatility for 10am expiration the next day trading at 9%. The calendar time corresponding to this implied volatility is 1/365: that is, the number of calendar days to tomorrow (1) divided by 365 days/year.

This means the variance is 0.092 \* 1/365 = 2.219x10-5.

This variance is unitless, and we can also identify it with trading time vol^2 \* trading time. Let’s choose to measure trading time in hours – it doesn’t matter much whether we choose hours or days or years for trading time units.

The trading time from 1pm today to 10am tomorrow (assuming the market is open the whole time) is 21 hours, plus 12 hours from the extra variance due to the 2-3pm event. So the trading time is 33 hours.

The trading time vol is then sqrt( variance / trading time ) = 8.20x10-3, in units of per square-root hours.

Now let’s move time to right after the event, and assume that trading time vol doesn’t change. Trading time has dropped significantly: it’s now 3pm, so two hours of normal time have passed; but also we’ve passed the event, so dropped another 12 trading time hours. The new trading time is 19 hours to expiration.

We can use the (unchanged) trading time vol and the new trading time to get the new variance, which equals trading time vol2 \* trading time = (8.20x10-3)2 x 19 = 1.278x10-5.

We can convert the variance to a market-convention implied volatility by using implied vol = sqrt( variance / calendar time). The calendar time is still 1/365, so the implied volatility changes to sqrt(1.278x10-5/ (1/365) ) = 6.83%.

So all else being equal, we expect the implied volatility to drop from 9.00% at 1pm, before the event, to 6.83% at 3pm, after the event.

**Question 2 (3 marks)**

In stochastic volatility models, why is there a smile? Describe the genesis of the smile in terms of vega gamma.

Similarly, describe why stochastic volatility models generate a skew, in terms of vega dspot.

Smile comes from volatility of volatility, paired with the symmetric vega gamma profile of vanilla options. If you buy a high-strike or a low-strike vanilla, you have a position that is long vega gamma and long vega. You can sell enough ATM vanilla to vega hedge the position; ATM options have zero vega gamma, so the position is still long vega gamma. Now, whichever way vol moves, you make money. That means traders tend to buy up high- and low-strike options and sell ATM options, which increases implied volatilities for high- and low-strike options vs the ATM implied volatility. That market pressure creates the implied volatility smile (symmetric because vega gamma is positive for both high- and low-strike options).

Skew comes from spot/vol correlation, paired with the asymmetric vega dspot profile of vanilla options. Assume spot/vol correlation is positive: if you buy a high-strike option, you have a position with long vega dspot and long vega. ATM options have zero vega dspot, so if you sell enough ATM vanilla to hedge vega, your position is long vega dspot and flat vega. Then if spot goes up, your vega turns positive; and you expect vol to go up because of the positive correlation, so you expect to make money. Similarly if spot goes down, vega turns negative right as you expect vol to go down because of the positive correlation, so you make money.

If you had bought a low-strike option instead, and put on a negative vega dspot position, you would lose money if spot goes up or down. This PNL behavior causes traders to buy high-strike options and sell low-strike options, creating a positive implied volatility smile.

If spot/vol correlation is negative, all the signs change, and traders want to sell high-strike options and buy low-strike options, creating a negative implied volatility smile.

So the sign of spot/vol correlation, along with the volatility of volatility (which determines the magnitude of correlated vol moves), paired with the asymmetric vega dspot profile, generates skew.

**Question 3 (2 marks)**

Why do most FX shops use a “sticky delta” volatility market model when defining delta for hedging purposes, even though that might not give the most accurate estimate of how implied volatilities, and hence portfolio prices, change when spot moves?

Risk managers need to define “axes” for their risk calculations: that is, which market data inputs they will treat as their main risk variables.

In the FX markets, implied volatility trades in the inter-dealer market in delta terms, so ATM vol, 25d risk reversal, 25d butterfly, 10d risk reversal, and 10d butterfly are the traded market variables.

Because of that, traders tend to look at portfolio risks that move on market input at a time: spot, keeping vol-by-delta constant; ATM vol, keeping spot and RR/BF constant; and RR/BF, keeping ATM vol and spot constant.

**Question 4 (4 marks)**

Consider an ATM EURGBP option with 0.5y to expiration. Assume the EURGBP ATM volatility is 3.5%, the EURUSD ATM volatility is 8.5%, and the GBPUSD ATM volatility is 7.5%. What is the implied correlation between EURUSD and GBPUSD spots?

EURUSD spot is 1.25 and GBPUSD spot is 1.56; assume zero interest rates.

Use the Black-Scholes vega formula to calculate the vegas of all three options and determine the notionals of EURUSD and GBPUSD options needed to hedge the vegas of 1 EUR notional of the EURGBP option, assuming correlation stays constant.

The cross volatility is defined in terms of correlation like

So we can calculate the correlation as

Plugging in the volatilities above, that gives a correlation of +91.18%.

To calculate the hedge notionals of the EURUSD and GBPUSD options, we need to start by finding how much the cross vol moves as each of the USD-pair vols move, assuming constant correlation (here we use index 1 to mean EURUSD, 2 to mean GBPUSD):

Then we need to determine what the vanilla options are. All three are ATM, so we can calculate the ATM strikes (assuming denominated premium currency for all three):

We can then use the Black-Scholes vega formula to calculate the vegas of all three options (denominated currency move per one unit asset currency of each):

Now we want to choose the notionals of EURUSD and GBPUSD options that hedge the move in the EURGBP option due to moves in EURUSD and GBPUSD vols, assuming the correlation does not move.

Let’s start with the EURUSD option. We can calculate the sensitivity of the GBP price of the EURGBP option to the EURUSD volatility as

Then we need to figure out what notional of the EURUSD option gives a vega that matches the number above; for this we need to remember that the vega calculated above for the EURUSD option is the USD price change per vol move, but we need GBP price change, and must convert the spot appropriately:

So for each unit EUR notional of the EURGBP option, you need to sell 0.7396 EUR of the EURUSD option to hedge its exposure to moves in EURUSD volatility, assuming correlation stays fixed.

Similarly we can calculate the notional of the GBPUSD option needed to hedge the sensitivity of the GBP price of the EURGBP option to moves in the GBPUSD volatility:

So for each unit EUR notional of the EURGBP option, you need to buy 0.0890 GBP of the GBPUSD option to hedge its exposure to moves in GBPUSD volatility, assuming correlation stays fixed.

**Question 5 (10 marks)**

In this question you will look at implied correlations and see how much moves in implied correlation contribute to moves in cross volatility, versus moves in the underlying USD-pair volatilities.

Consider the AUDJPY market, where the underlying USD pairs are AUDUSD and USDJPY.

For a given expiration tenor, one can calculate the market-implied correlation between moves in AUDUSD spot and USDJPY spot through the implied volatilities for the three pairs.

First step: write code to calculate these correlations in a window from 1Jan2007 to 31May2013. I have posted a spreadsheet with the ATM implied volatilities for AUDUSD, USDJPY, and AUDJPY for various expiration tenors on the class forum.

You should write a function that takes in the names of the three pairs (as strings like ‘AUDJPY’, ‘AUDUSD’, and ‘USDJPY’), a string tenor (like ‘3m’), a flag to define whether the cross spot is the product or the ratio of the two USD spots (which affects the sign of the correlation), and the start and end dates of the historical window.

It should start by loading the data for the ATM implied volatility for the three tenors from the spreadsheet into pandas DataFrames and then calculate a pandas DataFrame of implied correlations.

The next step: use the correlation from date i, along with the implied volatilities for the USD pairs on date i+1, to predict the cross volatility on date i+1. Do this with the pandas DataFrames you have already created.

Finally, construct two DataFrames: one holding day-to-day changes in the cross ATM volatility, and one holding differences between the predicted cross volatility (assuming the implied correlation from the day before) and the true cross volatility.

The function should print out statistics on both those series.

Run this for the following list of tenors: 1w, 1m, 6m, and 1y. Comment on any differences across tenors, and whether this seems like a good hedging strategy for hedging AUDJPY volatility. Make sure to refer to statistics of the two series, both standard deviations as well as maximum and minimum deviations.

Summarizing the results (code attached later):

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Tenor | Std Dev Hedged | Std Dev Unhedged | Max Hedged | Max Unhedged | Min Hedged | Min Unhedged |
| 1w | 1.19 | 1.80 | 14.95 | 18.81 | -13.92 | -15.88 |
| 1m | 0.74 | 1.10 | 7.87 | 12.55 | -7.31 | -8.63 |
| 6m | 0.38 | 0.58 | 5.88 | 4.92 | -3.42 | -3.72 |
| 1y | 0.32 | 0.43 | 4.61 | 3.72 | -3.27 | -2.48 |

“Std Dev Hedged” means the standard deviation of the estimated AUDJPY vol vs the true AUDJPY vol; “Std Dev Unhedged” means the standard deviation of the day-on-day moves of AUDJPY vol. Similarly for Max and Min values.

Some conclusions from the data set:

* The hedging works better for shorter tenors than longer tenors
* For intermediate tenors, standard deviation is reduced, but max/min moves are made worse
* Even for short tenors it doesn’t work that well: in 1w the standard deviation is reduced by only 1/3

To summarize, hedging AUDJPY vol moves assuming that the implied correlation doesn’t move day to day isn’t a very effective hedging strategy.

That’s fine: AUDJPY is a pretty liquid options market and bid/ask spreads are relatively tight when you hedge AUDJPY vega directly. This sort of hedging strategy makes more sense for less liquid cross markets where liquidity is lower.

Python code:

import io

import numpy

import pandas

def corr\_analysis(pairx, pair1, pair2, spots\_divided, ten, file\_name):

'''Runs an analysis on how well a constant-correlation assumption predicts moves

in cross vol.

pairx: string name of the cross pair (eg AUDJPY)

pair1: string name of the first USD pair (eg AUDUSD)

pair2: string name of the second USD pair (usd USDJPY)

spots\_divided: True if the cross spot is the ratio of the USD spots; False if it's the product

ten: implied volatility tenor

file\_name: string name of the file with the vol data

'''

# pull in the vol data via panda's read\_excel function

vols\_df = pandas.read\_excel(file\_name, index='Date')

# the columns are named like <pair> <ten> - define the column names for the pairs we care about

colx = pairx + ' ' + ten

col1 = pair1 + ' ' + ten

col2 = pair2 + ' ' + ten

# calculate the implied correlation. If the cross spot is the ratio of the two spots, the cross

# vol^2 = vol1^2 + vol2^2 - 2 corr vol1 vol2; if the cross spot is the product, the sign in front

# of the correlation term is positive.

ser\_corr = (vols\_df[colx]\*\*2 - vols\_df[col1]\*\*2 - vols\_df[col2]\*\*2) / 2. / vols\_df[col1] / vols\_df[col2]

if spots\_divided: ser\_corr \*= -1

# for each date, calculate the predicted cross vol, based on the previous date's correlation

# and the new USD vols

serx\_est = numpy.sqrt(vols\_df[col1]\*\*2 + vols\_df[col2]\*\*2 + 2 \* ser\_corr.shift(1) \* vols\_df[col1] \* vols\_df[col2])

# generate a series based on the differences between the estimated cross vol and the actual cross vol

diffs\_est = serx\_est - vols\_df[colx]

# then generate a series based on the daily-on-day differences in the cross vol as a reference

diffs\_1d = vols\_df[colx] - vols\_df[colx].shift(1)

# print out a little report showing the stats on the two series

print('Statistics for estimated cross vol minus realized cross vol')

print('-----------------------------------------------------------')

print(diffs\_est.describe())

print()

print('Statistics for day on day changes in cross vol')

print('----------------------------------------------')

print(diffs\_1d.describe())

def test():

'''Calls corr\_analysis with test data'''

pairx = 'AUDJPY'

pair1 = 'AUDUSD'

pair2 = 'USDJPY'

spots\_divided = False

tens = ['1w', '1m', '6m', '1y']

file\_name = '/home/mark.higgins.washingtonsquaretech.com/Uploads/fx\_vol\_data.xlsx'

for ten in tens:

print('For tenor', ten)

corr\_analysis(pairx, pair1, pair2, spots\_divided, ten, file\_name)

print()

print()

if \_\_name\_\_=='\_\_main\_\_':

test()